Modeling of a Power Transformer Using the Perturbation Finite Element Method

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Abstract — In this paper the perturbation technique is developed for modeling a power transformer. The paper analyses the influence of frames on the magnetic field behaviour and on the core losses calculation. Two types of frames are considered: frame manufactured with steel and frame of the steel with a copper layer. The model considers the eddy currents in copper layer of the frame and its effects on the magnetic field behaviour of the core.

I. INTRODUCTION

A transformer is a static device that transfers electrical energy from one circuit to another by electromagnetic induction without frequency change. There is a continuous increase in ratings of generator transformers and autotransformers. With the increase in MVA ratings, the weight and size of large transformers approach or exceed transport and manufacturing capability limits. Also, due to the ever increasing competition in the global market, there are continual efforts to optimize the material content in transformers [1].

With the development of numerical methods such as finite element method (FEM), modeling of transformers is now easier and less complicated. Some of the complex 3-D problems when solved by using 2-D formulations lead to significant inaccuracies. Nevertheless, 3-D FEM analysis may require considerable amount of time and computational efforts. Hence, wherever possible, a transformer designer would prefer fast analysis with sufficient accuracy so as to enable him to decide to perform changes in the project.

The perturbation technique is herein developed for modeling a power transformer. The core and frames of this transformer are showed in Fig. 1.

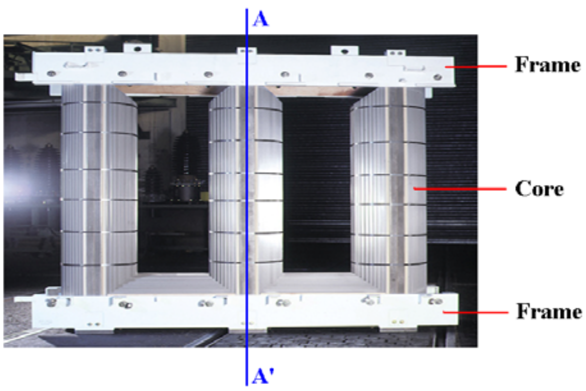

Fig. 1. The core and frames of the transformer analyzed.

This paper analyses the influence of frames on the magnetic field behaviour and on the core losses calculation. Two types of frames are considered: frame manufactured with steel and frame of the steel with a copper layer (see zoom of Fig. 2 (*left*)). The model considers the eddy currents in copper layer of the frame and its effects on the magnetic field behaviour of the core.

The full problem is tackled iteratively starting from a reference problem with a finite element (FE) solution. This solution is then modified iteratively when adding the magnetic frame to the initial configuration. Our reference problem is constituted by a core, windings carrying a sinusoidal current, magnetic shunt and the tank walls (Fig. 2 (*left*), transversal cut on the line AA′ showed in Fig. 1). A 2-D FEM is used and it enables the assessment of the effects due to intricate structural details such as the inclusion of the magnetic frame in front of the core. The use of a perturbation technique [2] allows accounting for any variation of geometrical or physical properties while avoiding a completely new FE computation, given that the solution of the reference model remains the same. Fig. 2 (*right*) shows the calculation domain and its 2-D mesh.

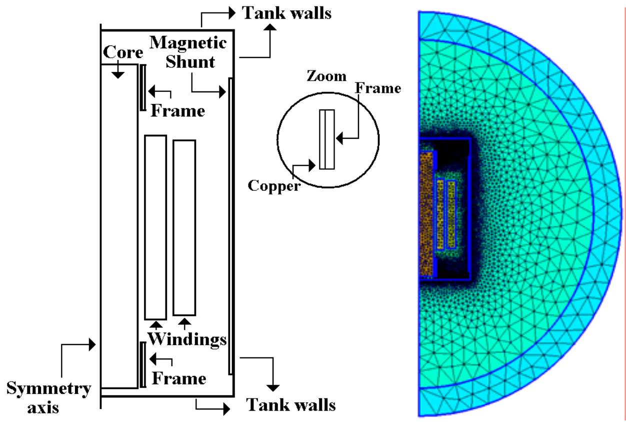

Fig. 2. Transversal cut on the line AA′ (*left*) and calculation domain and its 2-D mesh (*right*).

The first perturbation problem comprises, in addition to the core and two windings, a magnetic shunt and a tank. Its cross section in the XY plane (transversal cut on the line AA′ showed in Fig. 1) defines an initial 2-D model, to be further modified toward a 3-D model. This 2-D solution is considered invariant in the Z direction up to a certain distance. Beyond this distance, the magnetic field is chosen to be zero, which results in a particular interface condition to be further corrected. Then, this solution serves as source for a second perturbation problem allowing magnetic leakage flux in 3-D. The 3-D model allows accurately calculating the magnetic field in the vicinity of the frame extremities, core, etc. A third perturbation problem considers the eddy currents on the copper layer in the frame.

II. MAGNETODYNAMIC FORMULATION

A canonical problem p consists in solving the magnetodynamic equations in a bounded domain Ω_p, with boundary $\Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p} = \partial\Omega_p$ (possibly at infinity), of the 2-D or 3-D Euclidean space [2]. The eddy current conducting part of Ω_p is denoted $\Omega_{c,p}$ and the non-conducting one $\Omega_{c,p}^C$, with $\Omega_p = \Omega_{c,p} \cup \Omega_{c,p}^C$. Massive conductors belong to $\Omega_{c,p}$. The subscript p of each object refers to the associated problem p.

The equations, material relations, boundary conditions (BCs) and interface conditions (ICs) of problem p are [2]

$$\operatorname{curl} \boldsymbol{h}_p = \boldsymbol{j}_p \,, \quad \operatorname{curl} \boldsymbol{e}_p = -\partial_t \boldsymbol{b}_p \,, \quad \operatorname{div} \boldsymbol{b}_p = 0 \,, \qquad \text{(1a-b-c)}$$

$$\boldsymbol{b}_p = \mu_p \, \boldsymbol{h}_p + \boldsymbol{b}_{s,p} \,, \quad \boldsymbol{j}_p = \sigma_p \, \boldsymbol{e}_p + \boldsymbol{j}_{s,p} \,, \qquad \text{(1d-e)}$$

$$\boldsymbol{n} \times \boldsymbol{h}_p|_{\Gamma_{h,p}} = 0 \,, \ \boldsymbol{n} \times \boldsymbol{e}_p|_{\Gamma_{e,p} \subset \Gamma_{b,p}} = 0 \,, \ \boldsymbol{n} \cdot \boldsymbol{b}_p|_{\Gamma_{b,p}} = 0 \,, \ \text{(1f-g-h)}$$

$$[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p} = \boldsymbol{j}_{su,p}, \ [\boldsymbol{n} \times \boldsymbol{e}_p]_{\gamma_p} = \boldsymbol{k}_{su,p}, \ [\boldsymbol{n} \cdot \boldsymbol{b}_p]_{\gamma_p} = \boldsymbol{b}_{su,p}, \ \text{(1i-j-k)}$$

where $\boldsymbol{h}_p$ is the magnetic field, $\boldsymbol{b}_p$ is the magnetic flux density, $\boldsymbol{e}_p$ is the electric field, $\boldsymbol{j}_p$ is the electric current density (including source and eddy currents), μ_p is the magnetic permeability, σ_p is the electric conductivity and $\boldsymbol{n}$ is the external unit normal to a boundary. As will be shown, fields $\boldsymbol{b}_{s,p}$ and $\boldsymbol{j}_{s,p}$ are source fields that will serve for the coupling of different subproblems. Note that (1b) is only expressed in $\Omega_{c,p}$, whereas it is reduced to the form (1c) in $\Omega_{c,p}^C$. Also (1g) is more restrictive than (1h). The notation $[\,\cdot\,]_\gamma = \cdot\,|_{\gamma^+} - \cdot\,|_{\gamma^-}$ expresses the discontinuity of a quantity through any interface γ (with sides γ^+ and γ^-), which is allowed to be non-zero. The associated surface fields $\boldsymbol{j}_{su,p}$, $\boldsymbol{k}_{su,p}$ and $\boldsymbol{b}_{su,p}$ may be either known or not, respectively for fixing constraints or post-processing results.

The objective is solving successive problems, the superposition of which gives the solution of a complete problem [2]. For the case of two subproblems, the complete solution is

$$\boldsymbol{h} = \boldsymbol{h}_1 + \boldsymbol{h}_2 \,, \quad \boldsymbol{b} = \boldsymbol{b}_1 + \boldsymbol{b}_2 \,, \quad \boldsymbol{j} = \boldsymbol{j}_1 + \boldsymbol{j}_2 \,, \quad \boldsymbol{e} = \boldsymbol{e}_1 + \boldsymbol{e}_2 \,,$$
$$\boldsymbol{j}_{su} = \boldsymbol{j}_{su,1} + \boldsymbol{j}_{su,2} \,, \ \boldsymbol{k}_{su} = \boldsymbol{k}_{su,1} + \boldsymbol{k}_{su,2} \,, \ \boldsymbol{b}_{su} = \boldsymbol{b}_{su,1} + \boldsymbol{b}_{su,2} \,. \quad \text{(2)}$$

As first step, a problem $p = 1$ of form (1) is defined and called reference or source problem. A problem $p = 2$ of same form (1) is then defined as a perturbation problem that results from a change of permeability or conductivity, from μ_1 to μ_2 or σ_1 to σ_2, in some subregions. It is defined in domain Ω_2, i.e. a modified form of Ω_1. For linear materials, the complete problem resulting from this perturbation has a solution with form (2) under the condition that the source fields in (1d-e) are given by

$$\boldsymbol{b}_{s,2} = (\mu_2 - \mu_1)\, \boldsymbol{h}_1 \,, \quad \boldsymbol{j}_{s,2} = (\sigma_2 - \sigma_1)\, \boldsymbol{e}_1 \,. \qquad \text{(3-4)}$$

This way the sum of all the equations and relations of (1) respectively for p = 1 and p = 2 gives exactly these of the complete problem. Nonlinear analyses can be classically treated inside each problem, with possible inter-problem iterations. The perturbation fields are still governed by the classical Maxwell equations (1a-b-c) whereas their associated material relations include now the additional volume sources (3) and (4). These sources usefully only occur in the modified regions [2]. At the discrete level, the meshes of both reference and perturbation problems can be significantly simplified, each problem asking for mesh refinement of different regions.

III. RESULTS

The example considered for validation of the proposed approach is shown in Fig. 2 (*right*). The magnetic flux lines: reference, perturbation and corrected solutions are showed in Fig. 3 as an example of preliminary results. The reference problem, Fig. 3(a), is constituted by a core, windings carrying a sinusoidal current, magnetic shunt and the tank walls. The sub-problem 1, Fig. 3(b), considers the insertion of frames in the calculation domain. The sub-problem 2, Fig. 3(c), considers the eddy currents due to the copper layer in the frames. Fig. 3(d) shows the corrected solution.

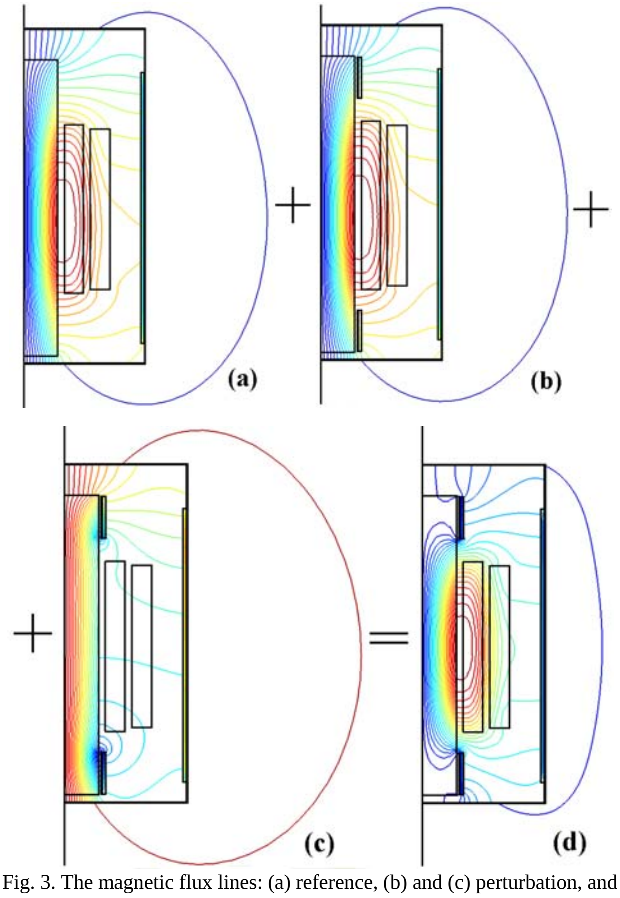

Fig. 3. The magnetic flux lines: (a) reference, (b) and (c) perturbation, and (d) corrected solutions.

The effects of 3-D model on the calculation of the magnetic field in the vicinity of the frame extremities, core, etc and the influence of frames on the transformer losses will be detailed and presented in the extended paper.

IV. REFERENCES

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